Machine Learning Assignment 50

Colby Roberts

October 10, 2020

50-1

Part 0

(A) The likelihood of $P(2\ 1\text{'s and }3\ 2\text{'}| \text{ switch})$

$$P(2\ 1\text{'s and }3\ 2\text{'s} \mid \text{switch}) = \begin{cases} \binom{5}{2} \cdot \frac{1}{2}^5 = 0.3125, & \text{switch} = \text{True} \\ \binom{5}{2} \cdot \frac{1}{6}^5 = 0.001286, & \text{switch} = \text{False} \end{cases}$$

Part 1

(A) The prior distribution of $P(\text{switch})$

$$P(\text{switch}) = \begin{cases} 50\%, & \text{switch} = \text{True} \\ 50\%, & \text{switch} = \text{False} \end{cases}$$

(B) The posterior distribution of $P(\text{switch} \mid 2\ 1\text{'s and }3\ 2\text{'s})$

$$P(\text{switch} \mid 2\ 1\text{'s and }3\ 2\text{'s}) = c \cdot P(2\ 1\text{'s and }3\ 2\text{'s} \mid \text{switch}) \cdot P(\text{switch}) = \begin{cases} \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.5 = 0.3125 \cdot 0.5, & \text{switch} = \text{True} \\ \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.5 = 0.001286 \cdot 0.5, & \text{switch} = \text{False} \end{cases}$$

$$P(\text{switch} \mid 2\ 1\text{'s and }3\ 2\text{'s}) = P(2\ 1\text{'s and }3\ 2\text{'s} \mid \text{switch}) = \begin{cases} c \cdot \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.5 = c \cdot 0.3125 \cdot 0.5 = 0.994, & \text{switch} = \text{True} \\ c \cdot \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.5 = c \cdot 0.001286 \cdot 0.5 = 0.006, & \text{switch} = \text{False} \end{cases}$$

$$\implies c \cdot (0.3125 + 0.001286) \cdot 0.5 = 1$$
$$\implies c = 3.1868852459 \cdot 2$$
$$\implies c = 6.3737704918$$

$$\implies P(\text{switch} \mid 2\ 1\text{'s and }3\ 2\text{'s}) = \begin{cases} 6.3737704918 \cdot \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.5 = 6.3737704918 \cdot 0.3125 \cdot 0.5 = 0.994, & \text{switch} = \text{True} \\ 6.3737704918 \cdot \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.5 = 6.3737704918 \cdot 0.001286 \cdot 0.5 = 0.006, & \text{switch} = \text{False} \end{cases}$$

Part 2

(A) The prior distribution of $P(\text{switch})$

$$P(\text{switch}) = \begin{cases} 1\%, & \text{switch} = \text{True} \\ 99\%, & \text{switch} = \text{False} \end{cases}$$

(B) The posterior distribution of $P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's})$

$$P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's}) = c \cdot P(2 \text{ 1's and } 3 \text{ 2's} \mid \text{switch}) \cdot P(\text{switch}) = \begin{cases} \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.01 = 0.3125 \cdot 0.01, & \text{switch} = \text{True} \\ \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.99 = 0.001286 \cdot 0.99, & \text{switch} = \text{False} \end{cases}$$

$$P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's}) = P(2 \text{ 1's and } 3 \text{ 2's} \mid \text{switch}) = \begin{cases} c \cdot \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.01 = c \cdot 0.3125 \cdot 0.01 = 0.994, & \text{switch} = \text{True} \\ c \cdot \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.99 = c \cdot 0.001286 \cdot 0.99 = 0.006, & \text{switch} = \text{False} \end{cases}$$

$$\implies c \cdot (0.3125 \cdot 0.01 + 0.001286 \cdot 0.99) = 1$$
$$\implies c = 227.368421053$$

$$\implies P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's}) = \begin{cases} 227.368421053 \cdot \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.01 = 227.368421053 \cdot 0.3125 \cdot 0.01 = 0.091374269, & \text{switch} = \text{True} \\ 227.368421053 \cdot \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.99 = 227.368421053 \cdot 0.001286 \cdot 0.99 = 0.006, & \text{switch} = \text{False} \end{cases}$$

Part 3

(A) The prior distribution of $P(\text{switch})$

$$P(\text{switch}) = \begin{cases} 99\%, & \text{switch} = \text{True} \\ 1\%, & \text{switch} = \text{False} \end{cases}$$

(B) The posterior distribution of $P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's})$

$$P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's}) = c \cdot P(2 \text{ 1's and } 3 \text{ 2's} \mid \text{switch}) \cdot P(\text{switch}) = \begin{cases} \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.99 = 0.3125 \cdot 0.99, & \text{switch} = \text{True} \\ \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.1 = 0.001286 \cdot 0.01, & \text{switch} = \text{False} \end{cases}$$

$$P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's}) = P(2 \text{ 1's and } 3 \text{ 2's} \mid \text{switch}) = \begin{cases} c \cdot \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.5 = c \cdot 0.3125 \cdot 0.99 = 0.994, & \text{switch} = \text{True} \\ c \cdot \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.5 = c \cdot 0.001286 \cdot 0.01 = 0.006, & \text{switch} = \text{False} \end{cases}$$

$$\implies c \cdot (0.3125 \cdot 0.99 + 0.001286 \cdot 0.01) = 1$$
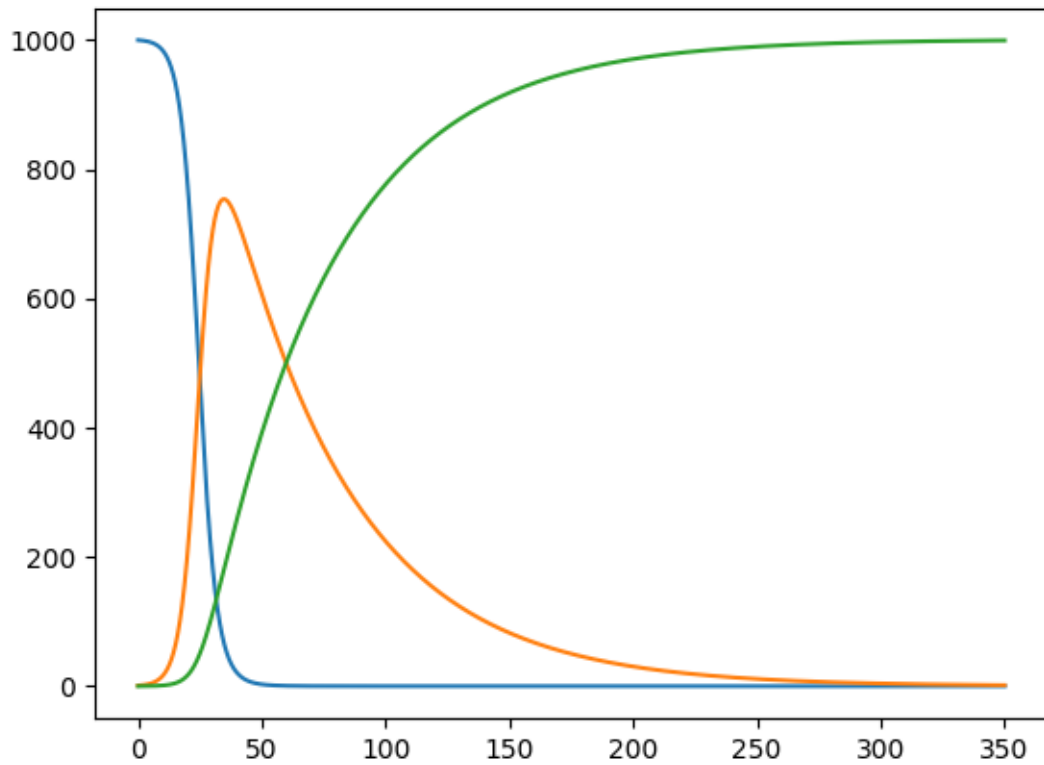$$\implies c = 3.23218887688$$

$$\implies P(\text{switch} \mid 2 \text{ 1's and } 3 \text{ 2's}) = \begin{cases} 3.23218887688 \cdot \binom{5}{2} \cdot \frac{1}{2}^5 \cdot 0.99 = 3.23218887688 \cdot 0.3125 \cdot 0.99 = 0.994, & \text{switch} = \text{True} \\ 3.23218887688 \cdot \binom{5}{2} \cdot \frac{1}{6}^5 \cdot 0.01 = 3.23218887688 \cdot 0.001286 \cdot 0.01 = 0.006, & \text{switch} = \text{False} \end{cases}$$

50-2

(A) system of differential equations to model the SIR system

$$\begin{cases} \frac{dS}{dt} = (-0.0003) \cdot SI, & S(0) = 1000 \\ \frac{dI}{dt} = (-0.02) \cdot I + (0.0003) \cdot SI, & I(0) = 1 \\ \frac{dR}{dt} = (0.02) \cdot I, & R(0) = 0 \end{cases}$$

(B)

(C) The blue line is the susceptible people, the orange line being the infected people and the green line is the recovered people. When there is a large amount of susceptible people the infected people increase exponentially, but after about 600 infected people the rate of infection drops less exponentially and the recovered people then explode exponentially to 1000 people while the population of susceptible people drops like a rock.