Assignment 63-1

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Part A

$$P(T \leq t) = \begin{cases} \frac{1}{16}t^2 & \text{for } 0 \leq t \leq 4 \\ 1 & \text{for } t > 4 \end{cases}$$

A

Find $P(T \leq 1)$

$$P(T \leq 1) = \frac{1}{16}(1)^2 = \frac{1}{16}$$

B

Find the probability that a job takes more than 2 hours

$$P(T > 2) = 1 - P(T \leq 2) = 1 - \frac{1}{16}(2)^2 = 1 - \frac{4}{16} = \frac{3}{4}$$

C

Find the probability that $1 \leq T \leq 3$

$$P(1 \leq T \leq 3) = P(T \leq 3) - P(T \leq 1) = \frac{1}{16}((3)^2 - (1)^2) = \frac{1}{16}(9 - 1) = \frac{1}{2}$$

Part B

if rainy traffic probability of 0.5
if not rainy traffic probability of 0.25

Rainy	Traffic	late to work
Yes	Yes	$\frac{1}{2}$
No	No	$\frac{1}{8}$
Yes	No	$\frac{1}{4}$
No	Yes	$\frac{1}{4}$

A

Probability of no rain heavy traffic and not late

Probability being late given no rain and heavy traffic is 0.25 and the probability of heavy traffic when it is not raining is 0.25 so the probability of both is $0.25 \times 0.25 = 0.0625$

B

Probability of late
We sum up the products of the path to each late. So It would be P(rain)*P(traffic—rain) * P(late—traffic rain) + P(rain)*P(no traffic—rain)*P(late—rain no traffic +
...

$$\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{8} =$$

$$\frac{1}{12} + \frac{1}{24} + \frac{2}{48} + \frac{6}{96} = \frac{8}{96} + \frac{4}{96} + \frac{4}{96} + \frac{6}{96} = \frac{22}{96} = \frac{11}{48}$$

C

Given that late probability of rain

$$P(rain|late) = \frac{P(rain\&late)}{P(late)} = \frac{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4}}{\frac{11}{24}} = \frac{\frac{1}{4} + \frac{1}{8}}{\frac{11}{24}} = \frac{24 \cdot 3}{11 \cdot 8} = \frac{9}{11}$$

Part C

Given a sample space

$$S = \{1, 2, 3, \dots\}$$

and a probability distribution

$$P(k) = P(\{k\}) = \frac{c}{3^k}$$

for $k = 1, 2, 3, \dots$

A

Find C

$$\sum_{n=1}^{\infty} \frac{c}{3^k} = 1$$

$$\sum_{n=1}^{\infty} c(\frac{1}{3})^k = \frac{c}{1 - \frac{1}{3}}$$

$$\frac{c}{1 - \frac{1}{3}} = 1 \rightarrow c = \frac{2}{3}$$

B

Find P(2,4,6)

$$P(\{2, 4, 6\}) = \frac{2}{3 \cdot 3^{(2)}} + \frac{2}{3 \cdot 3^{(4)}} + \frac{2}{3 \cdot 3^{(6)}} = \frac{2 \cdot 3^4}{3^7} + \frac{2 \cdot 3^2}{3^7} + \frac{2}{3^7} = \frac{182}{2187}$$

C

Find $P(\{3, 4, 5, \dots\})$

$$P(\{3, 4, 5, \dots\}) = \sum_{n=3}^{\infty} = \sum_{n=1}^{\infty} -P(2) - P(1) = 1 - \frac{2}{3 \cdot 3^2} - \frac{2}{3 \cdot 3} =$$

$$1 - \frac{6 + 2}{27} = \frac{19}{27}$$

Part D

An urn contains 30 red balls and 70 green balls. What is the probability of getting k red balls in a sample of size 20. with replacement.

P(red) = 3/10
P(green) = 7/10
Because we have replacement this probability is just the probability of getting k red balls in a sample size of 20 is

$$P(\text{k red balls in 20 picks}) = P(\text{k red and 20-k green}) = \binom{20}{k} (\frac{3}{10})^k \cdot (\frac{7}{10})^{20-k}$$

Part E

An urn contains 30 red balls and 70 green balls. What is the probability of getting k red balls in a sample of size 20. without replacement. Without replacement we cannot just multiply each time because after each pick the probability changes. So we can think of it this way. We will have k picks of the 30/100 probability and 20-k picks of the 70/100 probability and 20 pics total. SO we can model this as

$$\frac{(30)(29)\dots(30 - k)}{(100)(99)\dots(100 - k)} \cdot \frac{(70)(69)(70 - (20 - k))}{(100 - (k + 1))(100 - (k + 2))\dots(80)}$$

We can simplify this by using factorials:

$$\frac{\frac{30!}{(30-k)!} \cdot \frac{70!}{(70-(20-k))!}}{\frac{100!}{80!}} = \frac{80!30!70!}{(30-k)!(50+k)!(100)!}$$

Part F

$$P_X(x) = \begin{cases} 0.3 & x = 3 \\ 0.2 & x = 5 \\ 0.3 & x = 8 \\ 0.2 & x = 10 \\ 0 & \text{otherwise} \end{cases}$$

Find and Plot the CDF of X

)0.35)

Part G

Var(2X-Y) = 6 and Var(X+2Y) = 9. Find Var(X) and Var(Y) We need to use these 2 facts to solve this:

Var(X+Y) = Var(X)+Var(Y)+2Cov(X,Y) = Var(X)+Var(Y)*If X,Y are independant their covariance is 0

$Var(cX) = c^2 Var(X)$ Applying these to our equations we get.

Var(2X-Y) = Var(2X) - Var(Y) = 4Var(X) - Var(Y) $\rightarrow$ 4Var(X) - Var(Y) = 6

Var(X+2Y) = Var(X) + Var(2Y) = Var(X) + 4Var(Y) $\rightarrow$ Var(X) + 4Var(Y) = 9

We can then solve for Var(X) and Var(Y) using a matrix.

$$\begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} Var(X) \\ Var(Y) \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 & 6 \\ 1 & 4 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 9 \\ 4 & -1 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 9 \\ 0 & -17 & -30 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 9 \\ 0 & 1 & \frac{30}{17} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 9-\frac{4\cdot30}{17} \\ 0 & 1 & \frac{30}{17} \end{bmatrix}$$

$$Var(X) = \frac{153-120}{17} = \frac{33}{17}$$

$$Var(Y) = \frac{30}{17}$$